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High-Frequency Sum Rules for the Quasi-One-Dimensional Quantum Plasma Dielectric Tensor. II. Spin Effect

R. O. Genga[1]

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We derive a high-frequency expansion for all elements of the quasi-one-dimensional quantum plasma dielectric tensor at $T=0$ K for quantum particles with spins. In addition to the known results for spinless case, we find that $\Omega_{S,5}^{12}(\mathbf{k})$ and $\Omega_{S,5}^{23}(\mathbf{k})$ are the only frequency moments of the dielectric tensor with spin terms. Further, we find that there is no spin effect on quantum plasma dispersion for both ordinary and transverse modes propagating either along or across the external field.

1. INTRODUCTION

High-frequency sum rules of the full dielectric tensor in the absence and presence of external magnetic field for both classical nonrelativistic and relativistic plasmas and quantum nonrelativistic plasmas with spinless particles at $T=0$ K are known (Kalman and Genga, 1986; Genga, 1988*a–c*, 1989*a–c*). However, for a quantum nonrelativistic plasma with spin particles the existing work pertains to the application of the third moment to the electric and magnetic response function (Goodman and Sjonlander, 1973). The high-frequency sum rule is exact, but requires that $\Omega\omega^{-1} \ll 1$ and $\omega_p\omega^{-1} \ll 1$, where $\Omega = eB/mc$ and $\omega_p^2 = 4\pi e^2 n/m$.

In this work we consider the high-frequency sum-rule expansion to order ω^{-5} for the full dielectric tensor of quasi-one-dimensional quantum nonrelativistic plasmas with spin particles at $T=0$ K in regions where the external magnetic field is of the order of 10^{15} G, such as in pulsars. In these regions, we find that when the Fermi energy of electrons is lower than the excitation energy of the Landau levels, i.e., $p^2/2m \ll \hbar\Omega$, only the lowest

[1]Physics Department, University of Nairobi, P.O. Box 30197, Nairobi, Kenya.

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$n=0$ level is occupied, and the mobility of the electrons is therefore entirely determined by the value of the momentum along the z axis, i.e., $p=p_z$ (Canuto and Ventura, 1972; Genga, 1988*b,d*). This leads to a one-dimensional quantum plasma; this is realizable in situations where the particle density is low and the magnetic field is intense.

In Section 2 we review the derivation of the full dielectric tensor, whereas the exact ω^{-2}, ω^{-3}, ω^{-4}, and ω^{-5} sum-rule coefficients are calculated in Section 3. In Section 4 the long-wavelength aspect of the results of Section 3 is considered; the possible spin effects on high-frequency quantum plasma modes, i.e., the plasma mode and the high-frequency extraordinary mode for propagation parallel, perpendicular, and oblique to the magnetic field, respectively, are determined in Section 5.

2. DIELECTRIC TENSOR

As in the spinless situation, we treat an electron plasma in a constant homogeneous magnetic field quantum mechanically. While treating the magnetic field exactly, a perturbation approach in the photon field is used to derive the general expression for the dielectric tensor as (Canuto and Ventura, 1972; Pines and Nozières, 1966; Genga, 1988*b,c*)

$$\varepsilon^{\mu\nu}(\mathbf{k}0)=\delta^{\mu\nu}+\frac{\omega_p^2}{\omega^2}\mathrm{T}_{\mathbf{k}}^{\mu\nu}+\tilde{\alpha}^{\mu\nu}(\mathbf{k}\omega) \tag{1}$$

where

$$\tilde{\alpha}^{\mu\nu}(\mathbf{k}\omega)=-\frac{4\pi e^2}{\omega^2}\chi_s^{\mu\nu}(\mathbf{k}\omega) \tag{2}$$

$$\mathbf{T}_{\mathbf{k}}^{\mu\nu}=\delta^{\mu\nu}-\frac{k^\mu k^\nu}{k^2} \tag{3}$$

with

$$\chi_S^{\mu\nu}(\mathbf{k}\omega)=\sum_{nps}\langle 0,s|\Pi_{\mathbf{k}}^{\mu}(\tau)|s,n\rangle\langle n,s|\Pi_{\mathbf{k}}^{\nu}|s,0\rangle \times\left[\frac{1}{\omega-\omega_{n0}(p,p+\hbar k/2)+i\eta}-\frac{1}{\omega+\omega_{n0}(p,p-\hbar k/2)+i\eta}\right] \tag{4}$$

which is the spin current–current response tensor and

$$\Pi_k^\mu=\frac{1}{2m}\sum_i<[\Pi_i^\mu\exp(i\mathbf{k}\cdot\mathbf{x}_i)+\exp(i\mathbf{k}\cdot\mathbf{x}_1)\Pi_i^\mu] \tag{5}$$

$$\Pi_i^\mu=\mathrm{P}_i^\mu+eA^{0\mu}(x_i) \tag{6}$$

For the arguments of ω_{n0} and the summation over p in equation (4) we have, as in the spinless case, that

$$P = P_z, \qquad k = k_z \tag{7}$$

Consequently, for the summation over S in equation (4) we have

$$s = s_z \tag{8}$$

The matrix elements and excitation frequencies that appear in equation (3) are those appropriate for a system of electrons with Coulomb, spin, and external magnetic interactions, but without any transverse self-consistent magnetic field interactions.

3. SUM RULES

The complete modified polarizability tensor $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is expressible in terms of the corresponding "external" quantities $\hat{\bar{\alpha}}^{\mu\nu}(\mathbf{k}\omega)$ as (Genga, 1988*b*,*c*)

$$\bar{\alpha}(\mathbf{k}\omega) = \hat{\bar{\alpha}}(\mathbf{\Delta} - \hat{\bar{\alpha}})^{-1}\mathbf{\Delta} \tag{9}$$

where

$$\mathbf{\Delta} = \mathbb{1} - n^2\mathbf{T}, \qquad n^2 = \frac{kc}{\omega}, \qquad \mathbf{T} = \mathbb{1} - \frac{\mathbf{k}\cdot\mathbf{k}}{k^2} \tag{10}$$

It is known (Genga, 1988*b*,*c*; 1989*c*) that $\hat{\bar{\alpha}}^{\mu\nu}(\mathbf{k}\omega)$ possesses the high-frequency sum-rule expansion

$$\hat{\bar{\alpha}}^{H'\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=1\\ l\ \mathrm{odd}}}^{\infty} \frac{\Omega^{\mu\nu}_{s,l+1}(\mathbf{k})}{\omega^{l+1}} \tag{11}$$

$$\hat{\bar{\alpha}}^{H''\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=2\\ l\ \mathrm{even}}}^{\infty} \frac{\Omega^{\mu\nu}_{s,l+1}(\mathbf{k})}{\omega^{l+1}} \tag{12}$$

where the superscript H stands for "Hermitian part of"; the single and double primes denote "real part of" and "imaginary part of," respectively, and the $\hat{\Omega}^{\mu\nu}_s$ coefficients are obtained from equation (2) in the limit as $\eta \to 0$ to be of the form

$$\begin{aligned}\hat{\Omega}^{\mu\nu}_{s,l+1}(\mathbf{k}) = 4\pi e^2 \sum_{nps} \{&[\omega_{n0}(p, p - \hbar\mathbf{k}/2)]^{l-2} \\ &\times \langle 0, s|\Pi^{\mu}_{\mathbf{k}}(\tau)s, n\rangle \langle n, s|\Pi^{\nu}_{\mathbf{k}}|s, 0\rangle \\ &-[-\omega_{n0}(p, p + \hbar\mathbf{k}/2)]^{l-2}\langle 0, s|\Pi^{\nu}_{-\mathbf{k}}|s, n\rangle \\ &\times \langle n, s|\Pi^{\mu}_{\mathbf{k}}(0)|s, 0\rangle\}_{t=0}\end{aligned} \tag{13}$$

The high-frequency expansion of $\bar{\alpha}^{\mu\nu}(\mathbf{k})$ is known (Kalman, 1978; Kalman and Genga, 1986; Genga, 1988*a–c*) to be similar to that of $\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ given by equations (11) and (12), with $\bar{\Omega}^{\mu\nu}(\mathbf{k})$ replacing the corresponding $\hat{\Omega}_s^{\mu\nu}(\mathbf{k})$. The relationships between the two sets of coefficients up to $l=4$ are the same as for the spinless case, i.e.,

$$\begin{aligned}
\hat{\bar{\Omega}}{}^{\mu\nu}_{s,2}(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_{s,2} \\
\hat{\bar{\Omega}}{}^{\mu\nu}_{s,3}(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_{s,3} \\
\hat{\bar{\Omega}}{}^{\mu\nu}_{s,4}(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_{s,4} - \hat{\Omega}^{\mu\alpha}_{s,2}\hat{\Omega}^{\alpha\nu}_{s,2} \\
\hat{\bar{\Omega}}{}^{\mu\nu}_{s,5}(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_{s,5} - \hat{\Omega}^{\mu\alpha}_{s,2}\hat{\Omega}^{\alpha\nu}_{s,3} - \hat{\Omega}^{\mu\alpha}_{s,3}\hat{\Omega}^{\alpha\nu}_{s,2}
\end{aligned} \tag{14}$$

The Hamiltonian of the system that satisfies equation (13) is given by

$$\mathrm{H} = \sum_i \frac{\Pi^2}{2m^i} - \frac{\mu\hat{\mathbf{s}}\cdot\mathbf{B}}{2S} + \tfrac{1}{2}\sum_{i\neq j} V(\mathbf{x}_i - \mathbf{x}_j) \tag{15}$$

where $\hat{\mathbf{s}}$ is a unit spin vector, $\hat{\mathbf{B}} = \mathbf{B}^0/B^0$ is a unit magnetic field vector, $\mu = e\hbar/\mathrm{mc}$ is the magnetic moment, and $s = \frac{1}{2}\hbar$. The second term on the right-hand side of equation (15) is the interaction due to spins, whereas $V(\mathbf{x}_i - \mathbf{x}_j)$ is the interaction potential between a pair of particles and is independent of velocity.

Finally, with the above information, we now calculate the frequency moments (up to $l=4$). For an anisotropic system in the presence of an external magnetic field as in our case it is known (Genga, 1988*a,b*, 1989*b*) that $\bar{\alpha}^{\mu\nu}$ is nondiagonal; hence both even and odd moments of $\bar{\Omega}^{\mu\nu}_{l+1}$ exist. The real diagonal and off-diagonal elements of $\bar{\Omega}^{\mu\nu}_{s,l+1}$ satisfy the symmetry condition (Genga, 1988*a,b*, 1989*c*)

$$\hat{\bar{\Omega}}{}^{\mu\nu}_{s,l+1}(\mathbf{k}) = \hat{\bar{\Omega}}{}^{\nu\mu}_{s,l+1}(\mathbf{k}) \tag{16}$$

and the imaginary off-diagonal elements satisfy the antisymmetric condition

$$\hat{\bar{\Omega}}{}^{\mu\nu}_{s,l+1}(\mathbf{k}) = -\hat{\bar{\Omega}}{}^{\mu\nu}_{s,l+1}(\mathbf{k}) \tag{17}$$

The first moment is trivial,

$$\begin{aligned}
&\hat{\bar{\Omega}}{}^{\mu\nu}_{s,2}(\mathbf{k}) \\
&= 4\pi e^2 \sum_{nps}\left[\frac{\langle 0, s|\Pi^{\mu}_{\mathbf{k}}|n, s\rangle\langle n, s|\pi^{\nu}_{-\mathbf{k}}|s, 0\rangle}{\omega_{n0}(p, p+\hbar k/2)}\right. \\
&\quad \left. + \frac{\langle 0, s|\pi^{\nu}_{-\mathbf{k}}|n, s\rangle\langle n, s|\pi^{\mu}_{\mathbf{k}}|s, 0\rangle}{\omega_{n0}(p, p+\hbar k/2)}\right]_{t=0} \\
&= \hat{\Omega}^{\mu\nu}_2(\mathbf{k})
\end{aligned} \tag{18}$$

where (Genga, 1988*b*)

$$\hat{\hat{\Omega}}_2(\mathbf{k}) = \omega_p^2 \mathbf{L}^{\mu\nu}, \qquad L_k^{\mu\nu} = k^\mu k^\nu / k^2 \tag{19}$$

The second moment is given by

$$\begin{aligned}\hat{\hat{\Omega}}_{s,3}^{\mu\nu}(\mathbf{k}) &= 4\pi e^2 \sum_{nps} [\langle 0, s|\pi_{\mathbf{k}}^{\mu}|n, s\rangle \langle n, s|\pi_{-\mathbf{k}}^{\nu}|s, 0\rangle \\ &\quad - \langle 0, s|\pi_{\mathbf{k}}^{\mu}|n, s\rangle \langle n, s|\pi_{-\mathbf{k}}^{\nu}|s, 0\rangle]_{t=0} \\ &= 2\pi e^2 \langle 0, s|[\pi_{\mathbf{k}}^{\mu}, \pi_{-\mathbf{k}}^{\nu}] - [\pi_{-\mathbf{k}}^{\nu}, \pi_{\mathbf{k}}^{\mu}]|0, s\rangle|_{t=0} \\ &= \hat{\hat{\Omega}}_3^{\mu\nu}(\mathbf{k})\end{aligned} \tag{20}$$

where (Genga, 1988*b*)

$$\hat{\hat{\Omega}}_3^{\mu\nu} = i\omega_p^2 \frac{eB_\eta^0}{mc} \varepsilon^{\mu\eta\nu} \tag{21}$$

The third moment yields

$$\begin{aligned}\hat{\hat{\Omega}}_{s,2}^{\mu\nu}(\mathbf{k}) &= 4\pi e^2 \sum_{nps} \left[\omega_{n0}\left(p, p - \frac{\hbar k}{2}\right) \langle 0, s|\Pi_{\mathbf{k}}^{\mu}|n, s\rangle \langle s, n|\Pi_{-\mathbf{k}}^{\nu}|0, s\rangle \right. \\ &\quad \left. + \omega_{n0}\left(p, p + \frac{\hbar k}{2}\right) \langle 0, s|\Pi_{-\mathbf{k}}^{\nu}|n, s\rangle \langle s, n|\Pi_{-\mathbf{k}}^{\mu}|0, s\rangle \right]_{t=0} \\ &= 2\pi e^2 \langle 0, s[[\Pi_{\mathbf{k}}^{\mu}, \mathrm{H}], \Pi_{-\mathbf{k}}^{\nu}] + [[\Pi_{\mathbf{k}}^{\mu}, \mathrm{H}], \Pi_{-\mathbf{k}}^{\mu}]|0, s\rangle|_{t=0} \\ &= \hat{\hat{\Omega}}_4^{\mu\nu}\end{aligned} \tag{22}$$

where (Genga, 1988*b*)

$$\begin{aligned}\hat{\hat{\Omega}}_4^{\mu\nu}(\mathbf{k}) &= \omega_p^2 \frac{eB_\eta^0}{2m^2c} \hbar k^\alpha \langle 0, s| \varepsilon^{\mu\eta\nu} \frac{\partial}{\partial x^\alpha} + \varepsilon^{\mu\eta\alpha} \frac{\partial}{\partial x^\nu} + \varepsilon^{\alpha\eta\nu} \frac{\partial}{\partial x^\mu} \\ &\quad + i\varepsilon^{\mu\eta\nu}\varepsilon^{\alpha\eta\beta} \frac{eB_\eta^0}{2\hbar c} \chi^\beta|0, s\rangle - \omega_p^2 \frac{eB_\eta^0}{4m^2c} \hbar k^\mu \varepsilon^{\alpha\eta\nu} \langle s, 0| \frac{\partial}{\partial x^\alpha} \\ &\quad + i\varepsilon^{\alpha\eta\beta} \frac{eB_\eta^0}{2\hbar c} \chi^\beta|0, s\rangle - \omega_p^2 \frac{eB_\eta^0}{4m^2c} \hbar k^\nu \varepsilon^{\mu\eta\alpha} \langle s, 0| \frac{\partial}{\partial x^\alpha} \\ &\quad + i\varepsilon^{\alpha\eta\beta} \frac{eB_\eta^0}{2\hbar c} \chi^\beta|0, s\rangle - \omega_p^2 \frac{eB_\eta^0}{2m^2c} \hbar^2 k^\sigma k^\mu \langle s, 0|2mc(eB_\eta^0)^{-1} \frac{\partial}{\partial x^\alpha\, \partial x^\nu} \\ &\quad + i\varepsilon^{\alpha\eta\beta}\chi^\beta \frac{\partial}{\partial \chi^\nu} - i\varepsilon^{\alpha\eta\nu} \frac{\partial}{\partial \chi^\alpha} + \varepsilon^{\alpha\eta\mu}\varepsilon^{\alpha\eta\beta} \frac{eB_\eta^0}{2\hbar^2 c} (\chi^\beta)^2|0, s\rangle\end{aligned}$$

$$-\frac{\omega_p^2 eB_\eta^0}{2m^2c}\hbar^2 k^\alpha k^\nu \langle s,0|2mc(eB_\eta^0)^{-2}\frac{\partial^2}{\partial x^\alpha\,\partial x^\mu}-i\varepsilon^{\alpha\eta\mu}\chi^\alpha\frac{\partial}{\partial\chi^\alpha}$$

$$+i\varepsilon^{\alpha\eta\beta}\chi^\beta\frac{\partial}{\partial\chi^\mu}\varepsilon^{\alpha\eta\mu}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{2\hbar^2c}(\chi_\beta)^2|0,s\rangle-\frac{\omega_p^2 eB_\eta^0}{2m^2c}\hbar^2k^\alpha k^\alpha$$

$$\times\langle s,0|2mc(eB_\eta^0)^{-1}\frac{\partial^2}{\partial\chi^\mu\,\partial\chi^\nu}i\varepsilon^{\alpha\eta\beta}\chi^\beta\frac{\partial}{\partial\chi^\mu}i\varepsilon^{\alpha\eta\mu}\chi^\alpha\frac{\partial}{\partial\chi^\nu}$$

$$-\varepsilon^{\mu\eta\alpha}\varepsilon^{\beta\eta\nu}\frac{eB_\eta^0}{2\hbar^2c}\chi^\alpha\chi^\beta|0,s\rangle+\omega_p^4\langle s,0|\mathrm{L}_{\mathbf{k}}^{\mu\nu}+\frac{1}{N}\sum_{\mathbf{q}}\mathrm{L}_{\mathbf{q}}^{\mu\nu}(s_{\mathbf{k}-\mathbf{q}}-s_{\mathbf{q}})|0,s\rangle \tag{23}$$

The fourth moment leads to

$$\hat{\hat{\Omega}}_{s,5}^{\mu\nu}(\mathbf{k})=4\pi^2\sum_{nps}\left\{\left[\omega_{n0}\left(p,p-\frac{\hbar k}{2}\right)\right]^2\langle s,0|\Pi_{\mathbf{k}}^{\mu}(t)|n,s\rangle\langle s,n|\Pi_{-\mathbf{k}}^{\nu}|0,s\rangle\right.$$

$$\left.-\left[-\omega_{n0}\left(p,p-\frac{\hbar k}{2}\right)\right]^2\langle s,0|\Pi_{\mathbf{k}}^{\nu}|n,s\rangle\langle s,n|\Pi_{-\mathbf{k}}^{\mu}(t)|0,s\rangle\right\}_{t=0}$$

$$=2\Pi e^2\langle s,0|[[[\Pi_{\mathbf{k}}^{\mu},\mathrm{H}],\mathrm{H}],\Pi_{-\mathbf{k}}^{\nu}]-[[[\Pi_{-\mathbf{k}}^{\nu},\mathrm{H}],\mathrm{H}],\Pi_{\mathbf{k}}^{\mu}]|0,s\rangle|_{t=0}$$

$$=\hat{\hat{\Omega}}_5^{\mu\nu}(\mathbf{k})+\hat{\hat{\Omega}}_{s,5}^{\mu\nu *}(\mathbf{k}) \tag{24}$$

where

$$\hat{\hat{\Omega}}_5^{\mu\nu}(\mathbf{k})=\omega_p^2\frac{eB_\eta^0}{4m^3c^2}\hbar k\langle s,0|\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{(eB_\eta^0)^2}{4m\hbar c}\chi^\beta$$

$$+\tfrac{7}{4}\varepsilon^{\alpha\eta\mu}\pi^{\alpha\eta\beta}\varepsilon^{\omega\eta\beta}eB_\eta^0\chi^\beta\frac{\partial}{\partial\chi^\alpha}+\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\gamma\beta}\frac{(eB_\eta^0\chi^\beta)^2}{8m\hbar c}\frac{\partial}{\partial\chi^\mu}$$

$$+\varepsilon^{\mu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{(eB_\eta^0\chi^\beta)^2}{8m\hbar c}\frac{\partial}{\partial\chi^\nu}+i\varepsilon^{\mu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{(eB_\eta^0\chi^\beta)^3}{16m^2\hbar c^2}$$

$$+i6^{\mu\eta\alpha}\varepsilon^{\nu\eta\alpha}eB_\eta^0\frac{\partial}{\partial\chi^\alpha}|0,s\rangle+\frac{\omega_p^2 eB_\eta^0}{4m^2c}\hbar^2k^\alpha k^\mu\langle s,0|i\tfrac{7}{4}\varepsilon^{\alpha\mu\nu}\frac{\partial^2}{\partial\chi^\alpha\,\partial\chi^\alpha}$$

$$+\tfrac{17}{8}\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}eB_\eta^0\hbar^{-1}\chi^\beta\frac{\partial}{\partial\chi^\alpha}+i\tfrac{7}{4}\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{(eB_\eta^0\chi^\beta)^2}{m\hbar^2c^2}|0,s\rangle$$

$$+\frac{\omega_p^2 eB_\eta^0}{4m^2c}k^\alpha k^\mu\langle s,0|i\tfrac{7}{4}\varepsilon^{\alpha\eta\mu}\frac{\partial^2}{\partial\chi^\alpha\,\partial\chi^\alpha}+\tfrac{17}{8}\varepsilon^{\mu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{\hbar c}\chi^\beta\frac{\partial}{\partial\chi^\alpha}$$

$$+i\tfrac{7}{4}\varepsilon^{\mu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{(eB_\eta^0\chi^\beta)^2}{m\hbar^2c}|0,s\rangle+\frac{\omega_p^2eB_\eta^0}{4m^2c}k^\alpha k^\mu\langle s,0|i6\varepsilon^{\alpha\eta\mu}\frac{\partial^2}{\partial\chi^\alpha\,\partial\chi^\nu}$$

$$+i6\varepsilon^{\nu\eta\alpha}\frac{\partial^2}{\partial\chi^\alpha\,\partial\chi^\mu}+i\tfrac{3}{2}\,\varepsilon^{\nu\eta\mu}\frac{\partial^2}{\partial\chi^\alpha\,\partial\chi^\nu}+\tfrac{3}{2}\,\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{\hbar^2c}$$

$$+3\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{\hbar c}\chi^\beta\frac{\partial}{\partial\chi^\nu}+3\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{\hbar c}\chi^\beta\frac{\partial}{\partial x^\mu}$$

$$+i\tfrac{15}{4}\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\beta}\varepsilon^{\alpha\eta\beta}\frac{(eB_\eta^0\chi^\beta)^2}{m\hbar^2c^2}+i\varepsilon^{\mu\eta\nu}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{\hbar^2c}|0,s\rangle$$

$$+i\frac{\omega_p^2eB_\eta^0}{2m^2c}\langle s,0|\mathbf{L}_k^{\mu\nu}+\frac{1}{n}\sum_q(\varepsilon^{\mu\eta\alpha}\mathbf{L}_q^{\alpha\nu})(S_{\mathbf{k}-\mathbf{q}}-S_{\mathbf{q}})|0,s\rangle \tag{25}$$

is already known (Genga, 1988) and

$$\hat{\hat{\Omega}}{}^{\mu\nu*}_{s,5}(\mathbf{k})=-2\mu\frac{B^0\pi e^2}{m^2s}\langle s,0|\hat{\mathbf{s}}\cdot\hat{\mathbf{B}}\{[[\Pi_{\mathbf{k}}^\mu,\mathrm{H}],\Pi_{-\mathbf{k}}^\nu]+[[\Pi_{-\mathbf{k}}^\nu,\mathrm{H}],\Pi_{\mathbf{k}}^\mu]|0,s\rangle\}$$

$$=-\frac{2\mu B^0}{S}\hat{\Omega}_4^{\mu\nu}(\mathbf{k})\cos\theta$$

$$=2\Omega\hat{\hat{\Omega}}{}_4^{\mu\nu}(\mathbf{k})\cos\theta \tag{26}$$

is the term due to spins; θ is the angle between the external magnetic field B^0 and the spin vector $\mathbf{s}$, $\Omega=eB^0/mc$ is the electron cyclotron frequency, and $\Omega_4^{\mu\nu}(\mathbf{k})$ is the third frequency moment as given by equation (23). The factor of 2 in equation (26) is due to spin orientation. However, from the above we know that $\hat{\hat{\Omega}}{}_4^{\mu\nu}(\mathbf{k})$ are not coefficients of the imaginary off-diagonal elements and consequently vanish in this case, i.e.,

$$\hat{\hat{\Omega}}{}^{\mu\nu}_{s,5}(\mathbf{k})=0 \tag{27}$$

In order to obtain an explicit expression for $\bar{\Omega}_s^{\mu\nu}$, we choose the k-system, in which (Genga, 1988)

$$\mathbf{k}=(0,0,\mathbf{k}),\qquad \mathbf{B}^0=(B^0\sin\theta,0,\mathrm{B}^0\cos\theta) \tag{28}$$

(where θ is also the angle between the wave vector $\mathbf{k}$ and the external magnetic field $\mathbf{B}_0$), and

$$\mathbf{q}=(q\sin\theta\cos\theta,q\sin\theta\sin\theta,q\cos\theta) \tag{29}$$

The Landau gauge leading to the components of the magnetic field given in equation (28) is known (Genga, 1988*b*) to be of the form

$$\mathbf{A}^0=\tfrac{1}{2}(0,B_z^0x-zB_x^0,0) \tag{30}$$

4. LONG-WAVELENGTH LIMIT

The long-wavelength ($k \to 0$) limit of equations (19)–(26) leads to elements of the frequency moments of the form

$$
\begin{aligned}
&\hat{\Omega}^{11}_{s,2}(\mathbf{k}) = \hat{\Omega}^{22}_{s,2}(\mathbf{k}) = 0 \\
&\hat{\Omega}^{33}_{s,2}(\mathbf{k}) = \Omega \\
&\hat{\Omega}^{12}_{s,3}(\mathbf{k}) = -\Omega^{21}_{s,3}(\mathbf{k}) = i\omega_p^2 \Omega \cos\theta \\
&\hat{\Omega}^{23}_{s,3}(\mathbf{k}) = -\Omega^{32}_{s,3}(\mathbf{k}) = i\omega_p \Omega \cos\theta \\
&\hat{\Omega}^{11}_{s,4}(\mathbf{k}) = -\frac{2}{15}\frac{\omega_p^2}{m} E_{\text{corr}} k^2 \\
&\hat{\Omega}^{13}_{s,4}(\mathbf{k}) = 0 \\
&\hat{\Omega}^{22}_{s,4}(\mathbf{k}) = -\frac{2}{15}\frac{\omega_p^2}{m} E_{\text{corr}} k^2 \\
&\hat{\Omega}^{33}_{s,4}(\mathbf{k}) = \omega_p^4 - \frac{\omega_p^2}{m}\left(6E_F - \tfrac{4}{15}E_{\text{corr}}\right)k^2 \\
&\hat{\Omega}^{12}_{s,5}(\mathbf{k}) = -\hat{\Omega}^{12}_{s,5}(\mathbf{k}) = -\mathrm{i}\,\frac{\omega_p^2\Omega}{4m} + 3E_F + \tfrac{8}{15}E_{\text{corr}})k^2 \cos\theta \\
&\hat{\Omega}^{23}_{s,5}(\mathbf{k}) = -\hat{\Omega}^{32}_{s,5}(\mathbf{k}) = -\mathrm{i}\,\frac{\omega_p^2\Omega}{4m} + 15E_F + \tfrac{12}{15}E_{\text{corr}})k^2 \cos\theta
\end{aligned}
\tag{31}
$$

where

$$E_F = \frac{P_F^{02}}{2m} \tag{32}$$

and

$$E_{\text{corr}} = \frac{n}{v}\sum_q \frac{4\pi e^2}{q^2} g_q$$

is the correlation energy, which is negative.

From equation (31) we find that there is no spin effect in $\hat{\Omega}^{\mu\nu}_{s,n}$. Hence, we conclude that there is no spin effect on high-frequency nonrelativistic quantum plasma modes propagating along and across the magnetic field, respectively.

REFERENCES

Canuto, V., and Ventura, J. (1972). *Astrophysics and Space Science*, **18**, 104.
Genga, R. O. (1988*a*). *International Journal of Theoretical Physics*, **27**, 819.
Genga, R. O. (1988*b*). *International Journal of Theoretical Physics*, **27**, 851.
Genga, R. O. (1988*c*). *Kenya Journal of Sciences*, **9**, 47.
Genga, R. O. (1988*d*). *International Journal of Theoretical Physics*, **27**, 835.
Genga, R. O. (1989*a*). *International Journal of Theoretical Physics*, **28**, 71.
Genga, R. O. (1989*b*). *International Journal of Theoretical Physics*, **28**, 893.
Genga, R. O. (1989*c*). *Discovery of Innovation*, **1**, 38.
Goodman, B., and Sjonlander, A. (1973). *Phys. Rev.* **8**, 200.
Kalman, G. (1978). Methods and approximations for strongly coupled plasmas, in *Strongly Coupled Plasmas*, G. Kalman, ed., Plenum Press, New York.
Kalman, G., and Genga, R. (1986). *Physical Review A*, **33**, 604.
Pines, D., and Nozières, P. (1966). *The Theory of Quantum Liquids*, Benjamin, New York.